Population Bomb Damage Calculations

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1 Introduction

Population Bomb (PB) is a unique move given to Maushold in Pokemon Scarlet and Violet. The move has 20 base power, and can hit up to 10 times. Unlike other multi-hit moves such as Bullet Seed or Arm Thrust, the number of hits by PB is determined by when the individual attacks miss. PB continuously attacks its target with 90% accuracy bombs until it either misses or lands 10 times. This feature is not unique to PB - Triple Axel behaves quite similarly. However, PB has a much higher ceiling of damage. As a result, the optimal item choice of Maushold is an interesting study.

For many strong, but slightly inaccurate moves, common damage-boosting items such as Life Orb or Choice Specs/Band are preferred over accuracy-increasing items such as Wide Lens. The additional boost accuracy has always been outclassed by the raw power boost of other items. For PB, however, a 9% accuracy boost from 90% to 99% does not directly correlate with a 9% damage increase. Exactly how much it increases the average damage is not entirely clear. Here, we will discuss the ramifications of Maushold holding a Wide Lens and compute how it affects the damage calculations of PB.

2 Mathematical Discussion

To recap, PB launches a series of bombs on a target with each having a 90% chance to hit. The attack finishes when 10 bombs have connected or one bomb has missed, whichever occurs first. To generalize our problem slightly, let us assume that each attack has a probability p of connecting and that we can hit a maximum of N times. PB is simply a special case of this setting with $p = 0.9$ and $N = 10$. We would like to examine how changes in p affect the damage output of our attack. The most natural way of doing so is to compute the expected number of bombs that land per attack. This will be directly correlated with the average damage done.

Setup

At first, it seems like our scenario is modeled by a geometric random variable where the expected value is already known. However, a key difference between our model and a geometric random variable is that we have a maximum number of hits of 10 whereas a geometric distribution assumes that we continue until a miss with no cap. We will have to compute the expected number of hits ourselves.

Let X be a random variable denoting the number of individual bombs that land during one attack. It should be clear that since each bomb is independent of the others, we have

$$P(X = n) = p^n(1 - p), n < N$$

and

$$P(X = N) = p^N.$$

That is, the only way we can hit exactly n times is if we land successfully on our first n attempts and then miss the next attempt (recall that the probability of landing is p and thus the probability of missing is $1 - p$). This is the case for $n < N$, but to get exactly N hits, we only need to land N times and do not need to miss afterward since we have hit the cap and cannot attack further.

Calculating at least n hits

Although it is not of immediate need, let us use this knowledge to calculate $P(X \leq n)$ for $n < N$. This is an important calculation for situations where one only needs 4 hits to finish off a Pokemon. Note that

$$P(X \leq n) = P(X = 0) + P(X = 1) + \cdots + P(X = n),$$

and we already have a simple expression for the left-hand side. Using our results from earlier, we can combine them to obtain

$$P(X \leq n) = \sum_{k=0}^{n} P(X = k) = \sum_{k=0}^{n} p^k(1 - p) = (1 - p)\sum_{k=0}^{n} p^k.$$

This is a great equation, but it can be simplified greatly. At this time, we would like to find an alternate expression for the geometric series $\sum_{k=0}^{n} p^k$. This is a standard result in mathematics, but we can derive it ourselves since we will need it later.

Let $a = \sum_{k=0}^{\infty} p^k$. This is not exactly what we are after, but it will help us out later. It follows that

$$\begin{aligned} a &= 1 + \quad && p + p^2 + p^3 + \ldots \\ ap &= && p + p^2 + p^3 + \ldots \end{aligned}$$

Thus, $a - ap = 1$. Solving for a, we obtain $a = \frac{1}{1-p}$. Using a similar technique, note that we can write

$$\sum_{k=0}^{n} p^k = \sum_{k=0}^{\infty} p^k - \sum_{n+1}^{\infty} p^k = \sum_{k=0}^{\infty} p^k - p^{n+1} \sum_{k=0}^{\infty} p^k = a - ap^{n+1} = \frac{1-p^{n+1}}{1-p}$$

Here, we simply wrote the sum of the first k terms in the series as the difference between all of the terms in the series and all of the terms in the series except for the first k. From there, we can apply our definition of a and simplify the equation. We can then combine all our equations together to obtain

$$P(X \leq n) = (1-p) \sum_{k=0}^{n} = (1-p) \frac{1-p^{n+1}}{1-p} = 1 - p^{n+1}$$

which is much cleaner to look at. Consequently, we also have $P(X > n) = 1 - P(X \leq n) = p^{n+1}$. Using this, we can compute hypotheticals that were alluded to earlier. Suppose that Maushold needs more than 4 hits to finish off a weakened Pokemon. The probability this happens if Maushold is not holding an item is $0.9^5 \approx 0.59$, but if it is holding a Wide Lens, the probability is $(0.99)^5 \approx 0.96$ - a drastic improvement.

Calculating average hits

With this knowledge, we can now move on into calculating the average number of hits for every attack thrown. Since X is denoting the amount of hits of a single attack, this number is referred to as the expected value of X, or $E(X)$. The expected value is computed as a weighted sum of all possible outcomes. Indeed,

$$E(X) = \sum_{n=0}^{N} nP(X = n).$$

Thankfully, we already have a substitution for most of the left hand side

$$E(X) = \sum_{n=0}^{N-1} nP(X = n) + NP(X = N) = (1-p) \sum_{n=0}^{N-1} np^n + Np^N.$$

It now suffices to simplify $\sum_{n=0}^{N-1} np^n$. We can do this using a similar approach to the geometric series earlier. Let $b = \sum_{n=0}^{\infty} np^n$. Then it follows that

$$\begin{aligned} b = p+ \quad & 2p^2 + 3p^3 + \dots \\ bp = \quad & p^2 + 2p^3 + \dots \end{aligned}$$

similar to before. Thus, $b - bp = p \sum_{n=0}^{\infty} p^n$. Using our knowledge of geometric series from earlier, we conclude that $\sum_{n=0}^{\infty} np^n = b = \frac{p}{(1-p)^2}$. From here, we have the following

$$\begin{aligned}
\sum_{n=0}^{N-1} np^n &= \sum_{n=0}^{\infty} np^n - \sum_{n=N}^{\infty} np^n \\
&= \frac{p}{(1-p)^2} - p^N \left(\sum_{n=0}^{\infty} np^n + Np^n \right) \\
&= \frac{p}{(1-p)^2} - p^N \left(\sum_{n=0}^{\infty} np^n + N \sum_{n=0}^{\infty} p^n \right) \\
&= \frac{p}{(1-p)^2} - \frac{p^{N+1}}{(1-p)^2} - \frac{p^N N}{1-p} \\
&= \frac{p(1-p^N)}{(1-p)^2} - \frac{p^N N}{1-p}
\end{aligned}$$

after applying our previous calculations of the sums represented by a and b. We can then finally combine everything together to finally obtain

$$E(X) = (1-p) \left(\sum_{n=0}^{N-1} np^n \right) + Np^N = (1-p) \left(\frac{p(1-p^N)}{(1-p)^2} - \frac{Np^N}{1-p} \right) + Np^N = \frac{p(1-p^N)}{1-p}.$$

3 Final Results

We can now see the full impact of Wide Lens. With $p = 0.9$ and $N = 10$, the expected number of hits from PB comes just under 6 at around 5.8. This is still a massively powerful move, since the base power of each hit is 20 and technician is available as an ability. However, the difference of Wide Lens is remarkable. Substituting $p = 0.99$ instead, we get $E(X) \approx 9.5$. That is over 150% stronger than the itemless version. Put another way, Wide Lens is more impactful than a Choice Band on PB without any of the negative drawbacks. That's an average base power of 190 before STAB and technician boosts. Seeing as Maushold has very little offensive options outside of PB and many supporting moves, Wide Lens is drastically more viable than a Choice Band.